

HYPERBOLIC RATIONAL HOMOLOGY SPHERES NOT ADMITTING FILLABLE CONTACT STRUCTURES.

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ABSTRACT. In this paper, we exhibit an infinite family of hyperbolic rational homology 3–spheres which do not admit any fillable contact structures. We also note that most of these manifolds do admit tight contact structures.

1. INTRODUCTION

An important question in low–dimensional contact topology is whether a given closed, oriented, irreducible 3–manifold admits a tight contact structure or not. It follows from work of Gabai [10] that, if M is an irreducible 3–manifold and Σ is an oriented surface realizing a non-trivial homology class in M and of minimal genus among representatives of its homology class, then there is a taut foliation ξ of M with Σ as a leaf. In particular, Gabai can construct these taut foliations on any irreducible 3–manifold with $b_1(M) > 0$. Now it follows from work of Eliashberg-Thurston [4] that there is a fillable and hence tight (see discussion below) contact structure ξ' homotopic to ξ . Hence, we know that any irreducible 3–manifold with nontrivial second homology admits a tight contact structure. Work of Honda-Kazez-Matic [14] (and independently of Colin [3]) provides a tight contact structure on all toroidal 3–manifolds (regardless of homologically essential condition). With this, it follows from the work of Lisca and Stipsicz [20] that a Seifert fibered 3–manifold admits a tight contact structure if and only if it is not $(2q - 1)$ –surgery on the $(2, 2q + 1)$ –torus knot, for $q \geq 1$.

Since Seifert manifolds account for all geometric 3–manifolds other than hyperbolic ones, this leaves us with the following open question.

Question 1.1. *Does every hyperbolic 3–manifold admit a tight contact structure?*

From the discussion above, it suffices to restrict attention to hyperbolic rational homology spheres.

A related notion to tightness is that of a symplectic filling, whose definition we now recall.

Definition 1.2. A contact manifold (M, ξ) is *weakly fillable* if M is the oriented boundary of a symplectic manifold (X, ω) and $\omega|_{\xi} > 0$.

We remark that there are notions of strong and Stein fillability which in general are strictly stronger than the weak fillability that we have defined

here (see [5, 12]). For the rest of the paper fillable will stand for weakly fillable.

It follows from a theorem of Eliashberg and Gromov that if (M, ξ) is fillable then it is tight. It is also known that fillability is strictly stronger than tightness, see [6]. So we can refine the Question 1.1 and ask: does every hyperbolic 3-manifold which is a rational homology sphere admit a fillable contact structure? We answer this question negatively.

Theorem 1.3. *Let $M_{r,m}$ be the 3-manifold obtained by performing rational r -surgery along the pretzel knot $P(-2, 3, 2m + 1)$, $m \geq 3$ (see Figure 1). Then*

- (1) $M_{r,3}$ does not admit any fillable contact structure for $r \in [9, 15]$.
- (2) For m sufficiently large, there is a subinterval $[2m+3, s] \subset [2m+3, 2m + \frac{\sqrt{16m+13+9}}{2})$ such that $M_{r,m}$ does not admit any fillable contact structure for $r \in [2m+3, s]$.

Moreover, all of these manifolds do admit at least one tight contact structure for every $r \neq 2m + 3$.

It follows from the classification of exceptional surgeries on $P(-2, 3, 2m+1)$ that the manifolds $M_{r,m}$ are all hyperbolic for $m \geq 3$ and $r \neq 4m+6, 4m+7$ and $4m+8$, and three more exceptional surgeries $r = 16, 17$ and $\frac{37}{2}$ for $m = 3$. See [21], [28], [15] and [9]. Therefore, the theorem provides infinitely many examples of hyperbolic 3-manifolds that, while admitting tight contact structures, do not have any fillable contact structures.

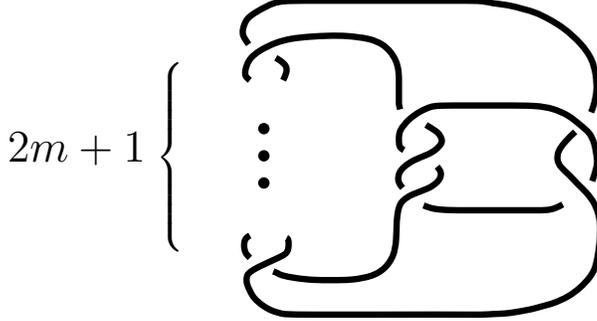


FIGURE 1. The pretzel knot $P(-2, 3, 2m + 1)$.

We want to make a couple of remarks about part (2) of Theorem 1.3.

- For any m , if there is a squarefree integer s in the interval $I_m = [2m+3, 2m + \frac{\sqrt{16m+13+9}}{2})$, then our proof of Theorem 1.3 shows that $M_{r,m}$ does not admit a fillable contact structure for any $r \in [2m+3, s]$. For example for $m = 4$, the interval $I_4 = [11, 17)$ contains three square-free integers, the largest of which is $s = 15$. Our argument in Section 3.2 shows that $M_{r,4}$ does not admit a fillable contact structure for any $r \in [11, 15]$. In general it is a delicate problem to guarantee

the existence of a squarefree integer in a given interval, but as we explain below, for m sufficiently large there is always at least one squarefree integer in the interval $(2m + 3, 2m + \frac{\sqrt{16m+13}+9}{2})$.

- The intervals of slopes mentioned in Theorem 1.3 are not necessarily the largest possible intervals. In fact we expect that for any surgery slope in the interval $[2m + 3, 4m + 6)$, the manifolds $M_{r,m}$ do not admit any fillable contact structures whenever $m > 3$ (for $m = 3$, due to surprising smaller exceptional surgeries on $P(-2, 3, 7)$, interval we expect is $[9, 17)$). See Section 3.2 for more details.
- Our goal in this note, motivated by Question 1.1, was to exhibit examples of hyperbolic rational homology spheres that do not have fillings and point out a curious subfamily of those non-fillable manifolds on which existence of a tight contact structure remains unclear (see Question 3.1).

We also want to mention that Etnyre-Baldwin in [1] proved that there are hyperbolic manifolds with tight non-fillable contact structures. It is however not known whether these manifolds admit any other fillable structures.

It was pointed out to us by John Etnyre that Youlin Li and Yajing Liu in [17] independently proved similar results.

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2. PRELIMINARIES

In this section we want to briefly recall some definitions and facts mainly from Heegaard Floer theory related to our knots $P(-2, 3, 2m + 1)$ and their surgeries.

2.1. L -space surgeries. Recall that a rational homology 3 sphere Y is called an L -space if $|H_1(Y; \mathbb{Z})| = \text{rk}(\widehat{HF}(Y))$. Here $\widehat{HF}(Y)$ is the hat version of Heegaard Floer homology group of Y .

A knot L in Y is called a L -space knot if a large enough positive surgery on L produces an L -space. It follows from work of Fintushel-Stern [8], Bleiler and Hodgson [2], and Ozsváth and Szabó [27] that $P(-2, 3, 2m + 1)$ pretzel knots as in our Theorem 1.3 are L -space knots. Moreover, for these knots it is known, by slice Bennequin bound in [23] and the maximal Thurston-Bennequin number calculation in [Theorem 6.1.4, [22]] that the slice genus denoted g_s is $m + 2 > 0$. Now a result of Lisca-Stipsicz in [Proposition 4.1, [18]] gives that the smooth 3-manifold $M_{r,m}$ obtained by

performing a rational r -surgery along pretzel knot $P(-2, 3, 2m + 1)$ is an L -space for every $r \geq 2g_s - 1 = 2m + 3$.

2.2. Negative-definite fillings. The following theorem, due to Ozsváth and Szabó, provides an obstruction to the existence of a symplectic filling for L -spaces.

Theorem 2.1 (Theorem 1.4, [26]). *If Y is an L -space, then all of its symplectic fillings are negative-definite.*

The first of the following theorems of Owens and Strle provides an obstruction for a certain 3-manifold to bound a negative-definite manifold. The second shows negative-definite fillings of a 3-manifold that is obtained by positive rational surgery do not have any “gap”.

Theorem 2.2 (Theorem 2, [24]). *Let Y be a rational homology 3-sphere with $|H_1(Y; \mathbb{Z})| = \delta$. If Y bounds a negative-definite four manifold X , and if either δ is square free or there is no torsion in $H_1(X; \mathbb{Z})$, then*

$$(2.1) \quad \max_{\mathfrak{t} \in \text{Spin}^c(Y)} 4d(Y, \mathfrak{t}) \geq \begin{cases} 1 - \frac{1}{\delta} & \text{for } \delta \text{ odd} \\ 1 & \text{for } \delta \text{ even} \end{cases}$$

Theorem 2.3 (Theorem 1(b), [25]). *For all rational numbers $r > m(K)$, the r -surgery $S_r^3(K)$ on K bounds a negative-definite manifold X_r , with $H^2(X_r) \rightarrow H^2(S_r^3(K))$ surjective where*

$$m(K) = \inf\{r \in \mathbb{Q}_{\geq 0} \mid S_r^3(K) \text{ bounds a negative-definite 4-manifold}\}.$$

3. PROOF OF THEOREM 1.3

In this section we give our proof of Theorem 1.3.

3.1. Tightness. Recall $M_{r,m}$ denotes a smooth rational r -surgery along the pretzel knot $P(-2, 3, 2m + 1)$. We prove that for any $r \neq 2m + 3$, $M_{r,m}$ does admit a tight contact structure. To this end, we first note that by work of Ng in [Theorem 6.1.4, [22]] the pretzel knot $P(-2, 3, 2m + 1)$ has the maximal Thurston-Bennequin number $\overline{tb}(P(-2, 3, 2m + 1)) = 2m + 3$. In particular, since the genus of $P(-2, 3, 2m + 1)$ is $m + 2$, we obtain that the knot type $P(-2, 3, 2m + 1)$ realizes the Bennequin bound. If the surgery coefficient $r < \overline{tb} = 2m + 3$, then, for each m , such surgery can be obtained by a contact surgery on a Legendrian link for which all contact surgery coefficients are negative, in particular it describes a Stein fillable contact structure, and hence it is tight. When $r > 2m + 3$, smooth r -surgery can also be obtained by a suitable contact surgery on a Legendrian link. It follows from work of Lisca-Stipsicz in [Theorem 1.1, [18]] that this contact surgery diagram describes a contact structure with non-zero Heegaard Floer contact invariant, and hence it is tight. For slope $r = 2m + 3$, there is an (obvious) corresponding contact surgery with contact framing zero, and such a contact surgery,

by definition, results an overtwisted contact structure. One then naturally wonders about the following question.

Question 3.1. *Does $M_{2m+3,m}$ also support a tight contact structure?*

Remark 3.2. We want to note the pretzel knots $P(-2, 3, 1)$, $P(-2, 3, 3)$ and $P(-2, 3, 5)$ are isotopic to the positive torus knots $T(2, 5)$, $T(3, 4)$, and $T(3, 5)$, respectively. Owens and Strle in [25] (cf. [Theorem 1.3, [16]]), extending results from [Theorem 4.2, [18]] found exact interval of surgeries on positive torus knots for which the surgery manifold does not admit fillable contact structures. On the other hand, there is exactly one surgery coefficient, $r = 3$ and only one of these knots, the torus knot $T(2, 5)$ (or $P(-2, 3, 1)$) for which the resulting surgery is known [20] to yield a manifold without any positive tight contact structures.

3.2. Fillability. In this section we study fillings of $M_{r,m}$. Our goal is to show that for any $P(-2, 3, 2m + 1)$ pretzel knot we can calculate a range of slopes, such that the surgeries with these slopes do not bound a negative-definite manifold. Indeed we will try to find largest possible interval such that for each integer in the interval, the inequality in Theorem 2.2 fails to hold.

We want to note $M_{4m+6,m}$ is orientation-preserving diffeomorphic to the small Seifert fibered space $M(-2, \frac{1}{2}, \frac{1}{4}, \frac{2m-7}{2m-5})$ which is a boundary of a negative-definite manifold for each $m \geq 3$, and all tight contact structures on $M(-2, \frac{1}{2}, \frac{1}{4}, \frac{2m-7}{2m-5})$ are Stein fillable (see [11]). Moreover as explained above that for any $r < 2m + 3$, $M_{r,m}$ does admit a Stein fillable contact structure. So necessarily the interval of surgeries that give non-fillable contact manifolds must be a subset of $[2m + 3, 4m + 6)$ for $m \geq 4$ (and $[9, 17)$ for $m = 3$, due to surprising smaller exceptional surgeries on $P(-2, 3, 7)$.)

3.2.1. Fillability of $M_{r,3}$. Let K denote the $P(-2, 3, 7)$ pretzel knot from now on. Its slice genus $g_s(K) = 5$. It follows from [Proposition 4.1,[18]] that the manifold obtained by any rational surgery $r \geq 9$ on K is an L -space. So we show that $M_{r,3}$ cannot bound a negative-definite 4-manifold for any $r \in (0, 15]$. Towards that end we use Theorem 2.2 and 2.3 of Owens and Strle.

We compute the d -invariant for the manifold obtained as 15-surgery on $P(-2, 3, 7)$ pretzel knot. For this we use the formula for d -invariants given in [Theorem 6.1,[24]] which we recall. If $L \subset S^3$ is an L -space knot and $t_i(L)$ denote its torsion coefficients. Then for $n > 0$ the d -invariants of the $\pm n$ -surgery on L are given by

$$d(L_n, i) = d(U_n, i) - 2t_i(L), d(L_{-n}, i) = -d(U_n, i)$$

for $|i| \leq n/2$ indexing a spin^c structure on L_n , where L_n denotes the manifold obtained by n -surgery along the knot L and U_n denotes manifold obtained by n -surgery along the unknot. The d -invariants for U_n are given

as

$$d(U_n, i) = \frac{(n - 2|i|)^2}{4n} - \frac{1}{4}$$

Recall that, if $L \subset S^3$ is an L -space knot and $\Delta_L(T) = a_0 + \sum_{j>0} a_j(T^j + T^{-j})$ is its symmetrized Alexander polynomial, then the torsion coefficients of L are given as

$$t_i(L) = \sum_{j>0} j a_{|i|+j}.$$

In the case of the $P(-2, 3, 7)$ pretzel knot K , the Alexander polynomial is given as

$$\Delta_K(T) = T^5 - T^4 + T^3 - T^2 + T - 1 + T^{-1} - T^{-2} + T^{-3} - T^{-4} + T^{-5}$$

So the torsion coefficients in this case are given as

$$t_0(K) = 3, t_{\pm 1}(K) = 2, t_{\pm 2}(K) = 2, t_{\pm 3}(K) = 1, t_{\pm 4}(K) = 1,$$

$$t_i(K) = 0 \text{ for } |i| \geq 5$$

So the d -invariants for $M_{15,3}$ are calculated as

$$d(M_{15,3}, i) = \begin{cases} -\frac{5}{2} & \text{if } i = 0 \\ -\frac{43}{30} & \text{if } |i| = 1 \\ -\frac{67}{30} & \text{if } |i| = 2 \\ -\frac{27}{30} & \text{if } |i| = 3 \\ -\frac{43}{30} & \text{if } |i| = 4 \\ \frac{1}{6} & \text{if } |i| = 5 \\ -\frac{1}{10} & \text{if } |i| = 6 \\ -\frac{7}{30} & \text{if } |i| = 7 \end{cases}$$

In particular for any $|i| \leq 7$, $4d(M_{15,3}, i) < \frac{14}{15}$, that is the inequality in Theorem 2.2 fails to hold for the squarefree surgery coefficient $r = 15$. Hence $M_{15,3}$ cannot bound a negative-definite 4-manifold. Now it follows from Theorem 2.3 that for any rational $0 < r \leq 15$, the surgered manifold $M_{r,3}$ cannot bound a negative-definite 4-manifold. Moreover, since $M_{r,3}$ is an L -space for any $r \geq 9$, by Theorem 2.1 we obtain that $M_{r,3}$ does not admit fillable structure for any $r \in [9, 15]$.

3.2.2. Fillability of $M_{r,m}$. We return to the general case. The idea of proof is already provided above. We just need to check some computational details. In what follows, we provide computations of torsion coefficients $t_i(P(-2, 3, 2m+1))$ in Lemma 3.4, then in Lemma 3.5 we provide an interval such that for each integer in the interval the corresponding surgery manifold fails to hold the inequality in Theorem 2.2.

The symmetrized Alexander polynomial for $P(-2, 3, 2m+1)$ is given by

$$\Delta_K(T) = T^{m+2} - T^{m+1} + \dots - T^{-(m+1)} + T^{-(m+2)}$$

This can be written as

$$\Delta_K(T) = a_0 + \sum_{j>0} a_j(T^j + T^{-j})$$

here $a_j = \pm 1$, for any j .

It can be seen easily from the formula for the Alexander polynomial that $a_i = (-1)^{i+m}$.

Following proposition will be used in the computation of the torsion invariants. Its proof is straightforward and is left for the reader as an exercise.

Proposition 3.3. *If $k > 1$ is an odd integer, then*

$$1 - 2 + 3 - 4 + \cdots + k = \frac{k+1}{2}.$$

If $k > 1$ is an even integer, then

$$-1 + 2 - 3 + \cdots + k = \frac{k}{2}.$$

Lemma 3.4. *Torsion coefficients, $t_i(K)$, for the $P(-2, 3, 2m+1)$ pretzel knot denoted by K are given as follows.*

(1) *If m is odd, then*

$$t_i(K) = \begin{cases} \frac{m+2-i}{2} & \text{if } i \leq m+2 \text{ odd} \\ \frac{m+3-i}{2} & \text{if } i < m+2 \text{ even} \\ 0 & i \geq m+2 \end{cases}$$

(2) *If m is even, then*

$$t_i(K) = \begin{cases} \frac{m+3-i}{2} & \text{if } i < m+2 \text{ odd} \\ \frac{m+2-i}{2} & \text{if } i \leq m+2 \text{ even} \\ 0 & i \geq m+2 \end{cases}$$

Proof. We only prove the first part. The second part is exactly analogous. Assume that $m \geq 3$ and odd. Torsion coefficients are defined as $t_i(K) = \sum_{j>0} j a_{|i|+j}$. If i is odd then

$$(3.1) \quad t_i(K) = a_{i+1} + 2a_{i+2} + \cdots + (m+2-i)a_{m+2}$$

$$(3.2) \quad = -1 + 2 - 3 + \cdots + (m+2-i)$$

It now follows from Proposition 3.3 that in this case $t_i(K) = \frac{m+2-i}{2}$ as $(m+2-i)$ is even. Similarly, if i is even, then

$$(3.3) \quad t_i(K) = a_{i+1} + 2a_{i+2} + \cdots + (m+2-i)a_{m+2}$$

$$(3.4) \quad = 1 - 2 + 3 - \cdots + (m+2-i)$$

Again by Proposition 3.3 we get that $t_i(K) = \frac{m+3-i}{2}$.

□

Now we compute the d -invariants for some surgeries. In particular, we show the following.

Lemma 3.5. *Let K denote the $P(-2, 3, 2m + 1)$ pretzel knot with $m \geq 3$. Let $M_{2k+m,m}$ denote the manifold obtained by $(2m + k)$ -surgery along K such that $k^2 - 9k < 4m - 17$. Then the inequality in Theorem 2.2 fails to hold.*

We first make some more general observations about the d -invariants of the surgered manifold.

Claim 3.6. The d -invariant $d(M, i)$ is negative for any $|i| < m + 2$ and $k^2 - 9k < 18m - 4$.

Proof. We compute the formula for d -invariant as

$$(3.5) \quad d(M, i) = \begin{cases} \frac{(2m+k-2|i|)^2}{4(2m+k)} - \frac{1}{4} - (m + 3 - i) & \text{if } m + i \text{ odd and } i < m + 2 \\ \frac{(2m+k-2|i|)^2}{4(2m+k)} - \frac{1}{4} - (m + 2 - i) & \text{if } m + i \text{ even and } i < m + 2 \end{cases}$$

To show that $d(M, i) < 0$ it suffices to show that the numerator of Equation 3.5 for each choice of m and i is negative. This will not be true in general. So some conditions on k are necessary. Assume m is odd and $i < m + 2$ is even. It is enough to check this for largest $i < m + 2$ odd. That is for $i = m + 1$. Simple calculation shows that the numerator is negative whenever $k^2 - 13k < 18m - 4$. The same calculation holds true when m is even and $i < m + 2$ is odd. Similarly one can show that when m is odd (or even) and $i < m + 2$ is odd (or even), then the numerator $(2m + k - 2|i|)^2 - (2m + k)(1 + 4(m + 3 - i))$ is negative whenever $k^2 - 9k < 18m$. So, by taking intersection of these we conclude that $d(M, i)$ is negative for any $|i| < m + 2$ and $k^2 - 9k < 18m - 4$. \square

Claim 3.7. The d -invariant $d(M, i)$ is a decreasing function for any $m + 2 \leq |i| \leq 2m + 2$. More importantly it assumes positive value for $i = m + 2$.

Proof. The proof is straightforward from the fact that the torsion coefficients $t_i(K) = 0$ for any $i \geq g(K) = m + 2$. \square

Proof of Lemma 3.5. Now combining claims above, we conclude that $d(M, i)$ attains its maximum value at $i = m + 2$ and it is calculated as $d(M_{2m+k,m}, m + 2) = \frac{(2m+k-2m-4)^2-2m-k}{4(2m+k)}$. Now for any integer N in the interval $[2m + 3, 2m + k]$ with $k^2 - 9k < 4m - 17$ (note such k 's satisfies $k^2 - 9k < 18m - 4$), it is easy to see that $4d(M_{2m+k,m}, m + 2) < \frac{2m+k-1}{2m+k}$ when k is odd and $4d(M_{2m+k,m}, m + 2) < 1$ when k is even. The proof follows. \square

Proof of Theorem 1.3. The fillability of $M_{r,3}$ and existence of tight contact structures on $M_{r,m}$ are already proved above. So, we are left to prove part (2) of the Theorem 1.3. Note that in Lemma 3.5, we found that for each integer N in the interval $[2m + 3, 2m + \frac{\sqrt{16m+13+9}}{2})$, the d -invariants of the

corresponding surgery manifold $M_{N,m}$ fail to satisfy the inequality in Theorem 2.2. In particular the same holds if there is a squarefree integer in the interval that Lemma 3.5 provides. There is much research done about the existence of a squarefree integer in a given interval. In particular work of Halberstam and Roth in [13] (cf [Theorem 1.1,[7]]) guarantees that for m sufficiently large there is always a squarefree integer, call it s , in the interval $(2m + 3, 2m + \frac{\sqrt{16m+13}+9}{2})$. Combining this with Theorem 2.3, we obtain that the desired conclusion of part (2) of Theorem 1.3. □

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